Lecture 5. Uncertainty in the Basic Model

Applied International Economics.

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The Model

In this model

Objectives

- Depending on household behavior (utility functions), consumption might vary when uncertainty arises in our simple model
- Understand the implications of self-insurance under uncertainty

Assumptions

- Endowment economy
- Two periods
- Single tradable good

Production and budget constraints

Suppose output in period 1 is given, and the endowment in period 2 is given by:

$$y_2 = \begin{cases} y_2^H \text{ with probability } p \\ y_2^L \text{ with probability } (1-p) \end{cases} \tag{1}$$

For the two periods we have

$$b_1 = y_1 - c_1, \tag{2}$$

$$0 = (1+r)b_1 + y_2^H - c_2^H, \tag{3}$$

$$0 = (1+r)b_1 + y_2^L - c_2^L, \tag{4}$$

with H being the high-output state and L the low output state.

Budget constraints

Combining the budget constraints yields the intertemporal budget constraints for each state of the economy:

$$y_1 + \frac{y_2^H}{1+r} = c_1 + \frac{c_2^H}{1+r} \tag{5}$$

$$y_1 + \frac{y_2^L}{1+r} = c_1 + \frac{c_2^L}{1+r} \tag{6}$$

Households

The households' utility function has the following form:

$$W = u(c_1) + \beta E\{u(c_2)\}, \tag{7}$$

$$W = u(c_1) + \beta[pu(c_2^H) + (1-p)pu(c_2^L)]$$

The first-order conditions (assuming $\beta(1+r) = 1$) are:

$$u'(c_1) = \lambda^H + \lambda^L, \tag{8}$$

$$pu'(c_2^H) = \lambda^H, \tag{9}$$

$$pu'(c_2^L) = \lambda^L. \tag{10}$$

Combining them we get:

$$u'(c_1) = pu(c_2^H) + (1-p)pu(c_2^L), \tag{11}$$

Which can be rewritten as:

$$u'(c_1) = E\{u'(c_2)\}, \tag{12}$$

this is called the *stochastic Euler equation*.

Solution of the model

The solution of the model will depend on the sign of u"'. Notice that u'(c) is linear, strictly convex or strictly concave depending on:

$$pu'(c_2^H) + (1-p)u'(c_2^L) = u'[pc_2^H + (1-p)c_2^L] \text{ if } u'''(c) = 0, \tag{13}$$

$$pu'(c_2^H) + (1-p)u'(c_2^L) > u'[pc_2^H + (1-p)c_2^L] \text{ if } u'''(c) > 0, \tag{14}$$

$$pu'(c_2^H) + (1-p)u'(c_2^L) < u'[pc_2^H + (1-p)c_2^L] \text{ if } u'''(c) < 0, \tag{15}$$

Case 1 (certainty equivalence) $u'''(c) = 0$

Suppose preferences are given by:

$$u(c) = -\frac{1}{2}(\bar{c} - c)^2 \tag{16}$$

Using our Euler equation (11), it follows that:

$$u'(c_1) = u'(E[c_2]). \tag{17}$$

Therefore,

$$c_1 = E[c_2]. \tag{18}$$

That is, households *smooth* consumption over time (given uncertainty).

Case 1 (certainty equivalence) $u'''(c) = 0$ (continued)

For a reduced form of c_1, multiply Equation (3) by p and Equation (4) by $(1 - p)$, and then sum them to obtain:

$$c_1 + \frac{E[c_2]}{1+r} = y_1 + \frac{E[y_2]}{1+r}. \tag{19}$$

Using the Euler equation we get:

$$c_1 = \frac{1+r}{2+r}\left[y_1 + \frac{E[y_2]}{1+r}\right]. \tag{20}$$

For the trade balance we have:

$$TB_1 = y_1 - c_1. \tag{21}$$

$$TB_1 = \frac{1}{2+r}\left(y_1 - E[y_2]\right). \tag{22}$$

Case 1 (certainty equivalence) $u'''(c) = 0$ (continued)

Assuming output is constant across periods (in an expected value sense), then

$$TB_1 = 0. \tag{23}$$

If the trade balance in period 1, even though the consumer faces uncertainty in period 2, it will not save in anticipation of this uncertainty. *There is no precautionary saving*. Now, using Equation (22) and Equations (3) and (4):

$$c_2^H = y_2^H + \frac{1+r}{2+r}\left[y_1 - E[y_2]\right], \tag{24}$$

$$c_2^L = y_2^L + \frac{1+r}{2+r}\left[y_1 - E[y_2]\right]. \tag{25}$$

Therefore, consumption in period 2 will depend on the realization of output.

Case 2 (more natural case): $u'''(c) > 0$

In this case consumers engage in precautionary savings as

$$u'(c_1) < u'(pc_2^H + (1-p)c_2^L), \tag{26}$$

which implies:

$$c_1 < E[c_2]. \tag{27}$$

Households want to save in case the bad state of the economy materializes! Naturally, since $TB_1 = CA_1$ and $CA_1 = y_1 - c_1 > 0$. **(Extra credit: Show this using Equation (19) and assuming again $y_1 = E[y_2]$.)** Therefore, in the presence of incomplete markets, uncertainty affects the economy's ability to smooth consumption over time because it induces precautionary saving.

What happens under complete markets?

Now suppose there are complete markets in the sense that the economy can buy state-contingent claims in world capital markets. This claim promises to pay one unit of output in the good state of nature as the price $q^H/(1+r)$, and one unit of output in the bad state of nature at the price $q^L/(1+r)$. As before, there also exists a risk-free asset that pays $1/(1+r)$. Therefore, by arbitrage:

$$\frac{1}{1+r} = \frac{q^H}{1+r} + \frac{q^L}{1+r}, \tag{28}$$

which simplifies to

$$1 = q^H + q^L. \tag{29}$$

Budget constraints

For period 1 we have (with b_1^H and b_1^L denoting the number of claims bought)

$$\frac{q^H}{1+r} b_1^H + \frac{q^L}{1+r} b_1^L = y_1 - c_1 \tag{30}$$

For period 2 in each of the states:

$$0 = b_1^H + y_2^H - c_2^H \tag{31}$$

$$0 = b_1^L + y_2^L - c_2^L \tag{32}$$

Combining, the intertemporal budget constraint becomes:

$$c_1 + \frac{q^H c_2^H + q^L c_2^L}{1+r} = y_1 + \frac{q^H y_2^H + q^L y_2^L}{1+r}. \tag{33}$$

Maximization problem

Using our new IBC, we obtain the following first-order conditions:

$$u'(c_1) = \lambda \tag{34}$$

$$pu'(c_2^H) = \lambda q^H \tag{35}$$

$$(1-p)u'(c_2^L) = \lambda q^L \tag{36}$$

Combining Equations (35) and (36) we have:

$$\frac{pu'(c_2^H)}{(1-p)u'(c_2^L)} = \frac{q^H}{q^L} \tag{37}$$

If we have actuarially fair prices (premiums pay the expected loss) we must have that:

$$\frac{q^H}{q^L} = \frac{p}{(1-p)}. \tag{38}$$

Which yields:

$$u'(c_2^H) = u'(c_2^L) \tag{39}$$

Consumers are allowed to smooth consumption over time.

In summary

- Complete markets allow households to smooth consumption over time
- With incomplete markets, output variability will affect welfare negatively, which is the usual case in developing economies.