Lecture 8. Two-Good Two-Sector Model and Relative Prices

Applied International Economics.

Gabriel Marin

Anahuac University, Spring 2024.

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Two-Good Two-Sector Model and Relative Prices

In this model

Objectives

- We know from the previous lesson that the relative price adjusts in response to various shocks when there are two goods
- Including two sectors instead of an endowed economy allows for sectoral reallocations of labor

Assumptions

- Two goods, tradable (foreign) and nontradable (home)
- Two sectors, with labor as only input
- Home good sector N is labor intensive
- Economy perfectly integrated with international capital markets

Budget constraints

The production function for each sector in each period t take the form:

$$y_t^T = Z^T (n_t^T)^\alpha, \qquad \text{for } t = 1, 2 \tag{1}$$

$$y_t^N = Z^N n_t^N, \qquad \text{for } t = 1, 2 \tag{2}$$

where y^T and y^N denote production of tradables and nontradables, respectively; Z^T and Z^N are (positive) technological parameters; and $0 < \alpha < 1$. We assume that the home goods (N) are labor intensive.

The labor supply constraint is the following:

$$n_t^T + n_t^N = \bar{n}, \qquad \text{for } t = 1, 2 \tag{3}$$

where $\bar{n}$ is the exogenously given supply of labor, which is inelastically supplied by the home and foreign goods sectors.

Budget constraints (continued)

For the two periods we have

$$b_1 = y_1^T + p_1 y_1^N - c_1^T - p_1 c_1^N, \tag{4}$$

$$0 = (1+r)b_1 + y_2^T + p_2 y_2^N - c_2^T - p_2 c_2^N, \tag{5}$$

where c_t^T and c_t^N represent the consumption of tradable and nontradables at time t, respectively; y_t^T and y_t^N are the endowments of tradable and nontradable goods, respectively. p represents the relative price of nontradable goods in terms of tradable goods.

Combining the budget constraints yields the intertemporal budget constraint (IBC)

$$y_1^T + p_1 y_1^N + \frac{y_2^T + p_2 y_2^N}{1+r} = c_1^T + p_1 c_1^N + \frac{c_2^T + p_2 c_2^N}{1+r}. \tag{6}$$

Substituting Equations (1) and (2) we obtain:

$$Z^T (n_1^T)^\alpha + p_1 Z^N n_1^N + \frac{Z^T (n_2^T)^\alpha + p_2 Z^N n_2^N}{1+r} = c_1^T + p_1 c_1^N + \frac{c_2^T + p_2 c_2^N}{1+r}. \tag{7}$$

Households

The households' utility function has the following form:

$$W = \gamma_1[log(c_1^T) + log(c_1^N)] + \beta\gamma_2[log(c_2^T) + log(c_2^N)], \tag{8}$$

where γ_t represents (positive) preference parameters for each period.

Households choose c_1^T, c_1^N, c_2^T, c_2^N, n_1^T, n_1^N, n_2^T, and n_2^N to maximize Equation (8) subject to Equation (7). The associated Lagrangian function is the following:

$$\begin{aligned}\mathcal{L} = \gamma_1[log(c_1^T) + log(c_1^N)] + \beta\gamma_2[log(c_2^T) + log(c_2^N)]+ \\ \lambda[Z^T(n_1^T)^\alpha + p_1 Z^N n_1^N + \frac{Z^T(n_2^T)^\alpha + p_2 Z^N n_2^N}{1+r} - c_1^T + p_1 c_1^N - \frac{c_2^T + p_2 c_2^N}{1+r}]+ \\ \mu_1[\bar{n} - n_1^T - n_1^N]+ \\ \frac{\mu_2}{1+r}[\bar{n} - n_2^T - n_2^N],\end{aligned} \tag{9}$$

The first-order conditions are (assuming $\beta(1+r) = 1$):

$$\frac{\gamma_1}{c_1^T} = \lambda, \tag{10}$$

$$\frac{\gamma_1}{c_1^N} = \lambda p_1, \tag{11}$$

Households (continued)

$$\lambda Z^T \alpha (n_1^T)^{\alpha-1} = \mu_1, \tag{12}$$

$$\lambda p_1 Z^N = \mu_1, \tag{13}$$

$$\frac{\gamma_2}{c_2^T} = \lambda, \tag{14}$$

$$\frac{\gamma_2}{c_2^N} = \lambda p_2, \tag{15}$$

$$\lambda Z^T \alpha (n_2^T)^{\alpha-1} = \mu_2, \tag{16}$$

$$\lambda p_2 Z^N = \mu_2. \tag{17}$$

If we combine Equations (10) and (11), and Equations (14) and (15) we obtain the *intratemporal* conditions for consumption:

$$c_1^T = p_1 c_1^N, \tag{18}$$

$$c_2^T = p_2 c_2^N. \tag{19}$$

Households (continued)

Now, combining Equations (12) and (13), and Equations (16) and (17) we obtain the production efficiency conditions:

$$\lambda Z^T \alpha (n_1^T)^{\alpha-1} = \lambda p_1 Z^N, \tag{20}$$

$$\lambda Z^T \alpha (n_2^T)^{\alpha-1} = \lambda p_2 Z^N. \tag{21}$$

At an optimum, households are equating the marginal productivity of labor across sectors. Should this not happen, then labor must reallocate to reach an optimum.

For further reference, combine Equations (20) and (21) to obtain:

$$\left(\frac{n_2^T}{n_1^T}\right)^{\alpha-1} = \frac{p_1}{p_2}. \tag{22}$$

Moreover, combine Equations (10) and (14) to obtain the Euler equation of tradable goods:

$$\frac{\gamma_1}{c_1^T} = \frac{\gamma_2}{c_2^T}. \tag{23}$$

By combining Equations (11) and (15) to obtain the Euler equation of nontradable goods:

$$\frac{\gamma_1}{p_1 c_1^N} = \frac{\gamma_2}{p_2 c_2^N}. \tag{24}$$

Equilibrium conditions

The consumption of nontradable goods must equal their fixed supply:

$$c_t^N = y_t^N, \quad \text{for } t = 1, 2. \tag{25}$$

Imposing the equilibrium in the nontradable goods market, we have that

$$b_1 = y_1^T - c_1^T, \tag{26}$$

$$-b_1 = rb_1 y_2^T - c_2^T, \tag{27}$$

which are the current account balances. Combining the two equations above, we obtain the economy's resource constraint:

$$y_1^T + \frac{y_2^T}{1+r} = c_1^T + \frac{c_2^T}{1+r}. \tag{28}$$

Finally, we have that the trade balance, as usual, is as follows:

$$TB_t = y_1^T - c_1^T, \quad \text{for } t = 1, 2. \tag{29}$$

Boom-bust cycle

Let us solve the model for the case when $\gamma_1 > \gamma_2$.

Key implications

- This simple model with production illustrates the dynamics of a boom-bust dynamics in a developing country: during the boom, consumption of both goods is high, there is a trade deficit, the relative price of non-tradables is high (real appreciation), production of tradables is low, and production of non-tradables is high.
- When the ”crisis“ comes, consumption of both goods collapses, the trade balance goes into surplus, the relative price of nontradables falls (real depreciation), and production shifts from the non-tradable sector to the tradable sector.